Ciphering Round Junior Varsity League

High School Math Competition 2008

Georgia Institute of Technology

February 23^{rd}, 2008

Problem #1

Problem

How many nonnegative integer solutions are there of the equation

$$5x + 11y = 90?$$

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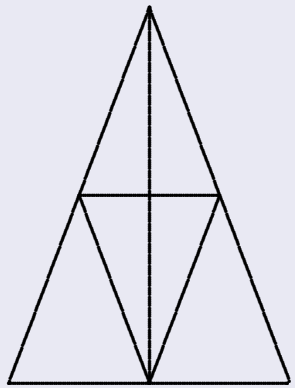
Answer

2 Solutions

Problem #2

Problem

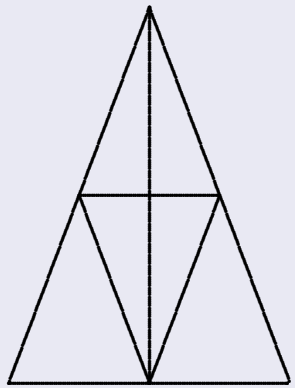
How many triangles are there in the following figure:



Problem #2

Problem

How many triangles are there in the following figure:



Answer

13

Problem #3

Problem

A swimmer is standing at a corner of a square swimming pool and she needs to get to the diagonally opposite corner. She can run at R miles per hour, and swim at S miles per hour. The swimmer can get to the opposite corner either by swimming directly across or by running around the perimeter. For what values of R (in terms of S) is it faster to run around the pool rather than swim across?

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Answer

$$R > \sqrt{2}S$$

Problem #4

Problem

Find the simplified value of

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Answer

$$\frac{61^2 - 39^2}{51^2 - 49^2} = \frac{(61 - 39)(61 + 39)}{(51 - 49)(51 + 49)} = \frac{22}{2} = 11$$

Problem #5

Problem

If $a, b, 10, c, d$ are in arithmetic progression, find $a + b + c + d$.

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If $a, b, 10, c, d$ are in arithmetic progression, find $a + b + c + d$.

Answer

$$a + b + c + d = 40$$

Problem #6

Problem

The sides of a right triangle are $\sqrt{3}$, $\sqrt{5}$ and x. Find the sum of all possible values for x.

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Answer

$$3\sqrt{2}$$

Problem #7

Problem

What is the last digit in the sum $3^{17} + 7^{13}$?

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Answer

0

Problem #8

Problem

How many numbers are there such that none of its digits is 0 and the sum of the digits is 6.

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Problem

How many numbers are there such that none of its digits is 0 and the sum of the digits is 6.

Answer

$$2^{6-1} = 32$$

Problem #9

Problem

Each of the numbers $1, 2, \ldots, 10$ is colored red or blue. 5 is red and at least one number is blue. If m and n are different colors and $m + n \leq 10$, then $m + n$ is blue. If m and n are different colors and $mn \leq 10$, then mn is red. What is the color of 7?

Problem #9

Problem

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Answer

Blue

Problem #10

Problem

Bob lives at (0,0) and works at (4,4). Every day, to get to work, he walks 8 blocks, (each block either increasing his x-coordinate or his y-coordinate). How many ways are there for him to get to work if the intersections (1,1) and (3,3) are closed off?

Problem #10

Problem

Bob lives at (0,0) and works at (4,4). Every day, to get to work, he walks 8 blocks, (each block either increasing his x-coordinate or his y-coordinate). How many ways are there for him to get to work if the intersections (1,1) and (3,3) are closed off?

Answer

$$14 = \binom{8}{4} - 2\binom{6}{3}\binom{2}{1} + \binom{4}{2}\binom{2}{1}^2$$

$$\tilde{T}(h_{\varepsilon}) \quad e^{n_d}$$