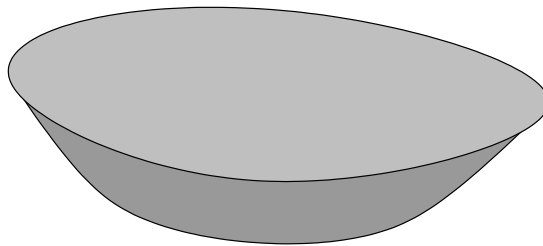


**2004 Georgia Tech High School Mathematics Competition**  
Varsity Multiple Choice – Version B

1. Solve  $|3x^2 - 2x - 1| \geq 0$ 
  - (a)  $x \geq 1$
  - (b)  $x \leq -1/3$
  - (c)  $x \leq -1$  or  $x \geq 1$
  - (d)  $1 \leq x \leq -1/3$
  - (e) none of the above
2. When  $x^{10} + 1$  is divided by  $x^2 - 1$ , the remainder is
  - (a) 1
  - (b)  $-1$
  - (c) 0
  - (d) 2
  - (e) none of these holds for every  $x$ .
3. Which of the following is logically equivalent to  $(P \text{ or } Q)$  and  $(Q \text{ or } R)$ ?
  - (a)  $P$  and  $R$
  - (b)  $(P \text{ and } R)$  or  $Q$
  - (c)  $(P \text{ and } Q)$  or  $(Q \text{ and } R)$
  - (d)  $(\text{Not } Q)$  and  $(P \text{ or } R)$
  - (e) None of the above
4. A  $1/50$  scale model of a pond is shown below. If the volume of the scale model is  $4 \text{ cm}^3$ , then the volume of the actual pond is
  - (a)  $1/2 \text{ m}^3$
  - (b)  $50000 \text{ cm}^3$
  - (c)  $500 \text{ m}^3$
  - (d) not determined by the given information
  - (e) none of the above

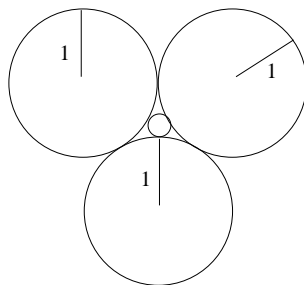


5. A number system based on 26 is used and the letters of the alphabet are the digits:  $A = 0, B = 1, C = 2, D = 3, \dots, X = 23, Y = 24, Z = 25$ . In this system, how much is ONE + ONE?

- (a) DBIA    (b) BAID    (c) BDAI    (d) DAIB    (e) BDIA

6. The radius of the small circle in the middle is

- (a)  $\frac{1}{6}$     (b)  $\frac{1}{8}$     (c)  $\frac{1}{4\sqrt{3}}$     (d)  $\frac{\sqrt{3}-1}{4}$   
 (e)  $\frac{2\sqrt{3}-3}{3}$     (f)  $\frac{2\sqrt{3}+1}{8}$     (g)  $\frac{2\sqrt{3}}{9}$



7. A function  $f : X \rightarrow Y$  is called one-to-one if for  $x_1, x_2 \in X$ ,  $f(x_1) = f(x_2)$  implies  $x_1 = x_2$ . Define

$$\begin{aligned} f : [-2, 0] &\rightarrow \mathbb{R}, & f(x) &= \frac{x+1}{x-1} \\ g : [0, 3] &\rightarrow \mathbb{R}, & g(x) &= (x+1)^2 \\ h : [-2, 1] &\rightarrow \mathbb{R}, & h(x) &= \sqrt{x^2+1} \\ k : [0, 3] &\rightarrow \mathbb{R}, & k(x) &= (1-\sqrt{x})^2 \end{aligned}$$

which of the following is true.

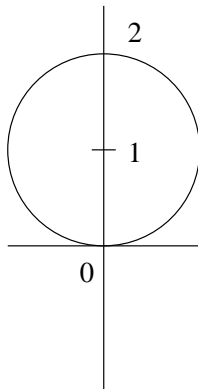
- (a)  $f$  and  $g$  are one-to-one functions  
 (b)  $g, h, k$  are one-to-one functions  
 (c)  $f, h, k$  are one-to-one functions  
 (d)  $f, k$  are one-to-one functions  
 (e)  $f, g, h, k$  are one-to-one functions

8. A drawer has 6 red socks and 6 white socks. If you reach in the drawer and randomly take out two socks, what is the chance (i.e., probability) that the two socks will match in color?

- (a)  $\frac{5}{12}$     (b)  $\frac{1}{2}$     (c)  $\frac{2}{5}$     (d)  $\frac{3}{7}$     (e)  $\frac{5}{11}$     (f)  $\frac{3}{8}$

9. Let  $z = x + iy = re^{i\theta}$ , which of the following equations corresponds to the circle graphed below?

- (a)  $r = 1 + \sin \theta$     (b)  $r = 2 + \sin \theta$     (c)  $r = 2 \cos \theta$   
(d)  $r = 2 \sin \theta$     (e)  $r = 2 - \cos \theta$

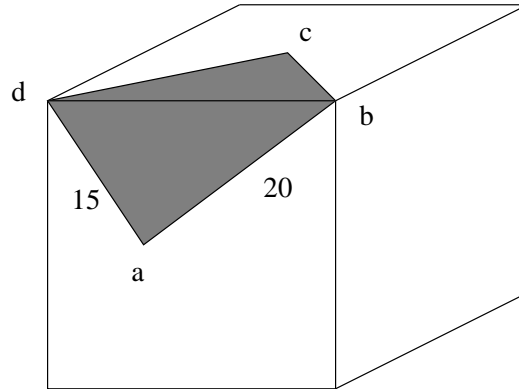


10. How many distinct solutions  $x$  are there for the equation  $\det(A - xI) = 0$  where  $\det$  stands for determinant,  $I$  is the identity matrix, and  $A$  is the matrix

$$\begin{bmatrix} 1 & 0 & -3 & 0 \\ 0 & 2 & 0 & 1 \\ 1 & 0 & 3 & 0 \\ 1 & -2 & 0 & 1 \end{bmatrix}$$

- (a) no solution    (b) 1    (c) 2    (d) 3    (e) more than 3

11. If a 15 by 20 rectangular piece of paper with corners  $a$ ,  $b$ ,  $c$ , and  $d$  is glued to the surface of a large cube so that the edge of the cube lies along the diagonal of the paper as shown, how far is corner  $a$  from corner  $c$  as measured through the cube?



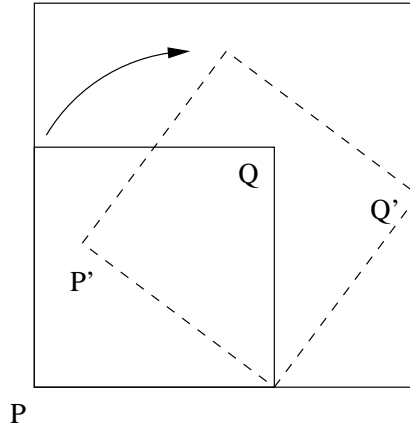
- (a)  $13\sqrt{2}$  (b) 18 (c) 18.2 (d)  $\sqrt{337}$  (e) None of the above
12. Find the quadratic equation whose roots are the cubes of the roots of

$$x^2 + ax + b = 0.$$

- (a)  $x^2 + a^3x + b^3 = 0$  (b)  $ax^2 + bx + ab = 0$   
(c)  $x^2 + (a^3 + b^3)x + ab = 0$  (d)  $x^2 + a(a^2 - 3b)x + b^3 = 0$   
(e) None of the above
13. If  $C$  is a circle with center  $(\sqrt{2}, \sqrt{3})$ , what is the maximum number of points  $(m, n)$  with integer coordinates that lie on  $C$ ?
- (a) 1 (b) 2 (c) 6 (d) 12 (e) no maximum

14. The equation  $\sqrt{x+5} - \sqrt{x-2} + 1 = 0$  has
- (a) one real root      (b) one real root and 1 complex root  
(c) 2 complex roots      (d) two real roots      (e) no root
15. Assume that  $\sin x + \cos x = a$ . Find  $\sin^4 x + \cos^4 x$  in terms of  $a$ .
- (a)  $a^4$       (b)  $a^2$       (c)  $a^4 + a^2 + 1$       (d)  $(1 + 2a^2 - a^4)/2$   
(e) none of the above
16. Three boys  $A$ ,  $B$ , and  $C$  are making a batch of cookies. It takes  $A$  working alone twice as long as it takes  $B$  working alone to make the same batch of cookies. It takes  $A$  four times as long as  $B$  and  $C$  working together to make the same batch. And it takes  $A$  2 hours longer to make the batch than it takes all three boys working together. How long will it take  $A$  and  $C$  to make the same batch of cookies?
- (a) 1 hr.      (b)  $1\frac{1}{2}$  hr.      (c)  $\frac{1}{6}$  hr.  
(d)  $\frac{5}{6}$  hr.      (e)  $\frac{3}{4}$  hr.
17. If  $11^{100} - 1 = 10^n \cdot m$  for integers  $m$  and  $n$  with  $m$  not divisible by 10, then what is  $n$ ?
- (a) 0      (b) 1      (c) 2      (d) 3      (e) 4
18. For  $-\frac{\pi}{4} < x < 0$ ,
- (I)  $\sin x < x$       (II)  $\sin x > x$       (III)  $\tan x < x$   
(IV)  $\tan x > x$       (V)  $\sin x > \cos x$       (VI)  $\sin x < \cos x$
- which of the following is true:
- (a) (II, III, VI)      (b) (II, IV, VI)      (c) (I, III, VI)  
(d) (I, IV, VI)      (e) (I, III, V)

19. A square of side length 5 rolls around the inside perimeter of a square of side length 8 as indicated in the figure. Eventually the corners  $P$  and  $Q$  will return to their original positions. If the first time this happens  $P$  has traveled a distance  $d$ , how far has  $Q$  traveled?



- (a)  $\frac{d-10\sqrt{2}}{1+\sqrt{2}}\pi$       (b)  $10 \tan^{-1}(3/4)$       (c)  $d/(2\sqrt{2})$   
 (d)  $10\pi + (\sqrt{2} + 1)(10\pi - d)$       (e) None of the above
20. A plane can be divided into 2 regions by one line, 4 regions by two lines, and 7 regions by three lines. What is the maximal number of regions that a plane can be divided into by 100 lines?
- (a) 451      (b) 851      (c) 1051      (d) 3051  
 (e) 4051      (f) 5051      (g) 6051      (h) 7051