2004 Georgia Tech High School Mathematics Competition

Junior Varsity Multiple Choice – Version B

1. Solve $|3x^2 - 2x - 1| \geq 0$

   (a) $x \geq 1$ (b) $x \leq -1/3$ (c) $x \leq -1$ or $x \geq 1$

   (d) $1 \leq x \leq -1/3$ (e) none of the above

2. $\sqrt{5/12} + \sqrt{1/5}$ $\square$ $\sqrt{1/3} + \sqrt{2/7}$

   What relation belongs in the box?

   (a) $<$ (b) $=$ (c) $>$ (d) $+$ (e) $-$

3. $\sqrt{7 + 4\sqrt{3}} - \sqrt{7 - 4\sqrt{3}} =$

   (a) 4 (b) $2\sqrt{3}$ (c) $\sqrt{6}$ (d) 2 (e) $2\sqrt{7}$

4. Solve for x: $3^x - 3^{-x} = 80/9$.

   (a) .8 (b) $\log_3\left(\frac{82}{19}\right)$ (c) 3/2 (d) $\frac{40-\sqrt{1681}}{9}$ (e) 2

5. A drawer has 6 red socks and 6 white socks. If you reach in the drawer and randomly take out two socks, what is the chance (i.e., probability) that the two socks will match in color?

   (a) $\frac{5}{12}$ (b) $\frac{1}{2}$ (c) $\frac{2}{5}$ (d) $\frac{3}{7}$ (e) $\frac{5}{11}$ (f) $\frac{3}{8}$

6. If m men can do a job in d days, then $m + r$ men can do the job in:

   (a) $d + r$ days (b) $d - r$ days (c) $\frac{md}{m+r}$ days

   (d) $\frac{d}{m+r}$ days (e) none of these

7. In how many ways can one give change for \$2.83 using only pennies and nickels?

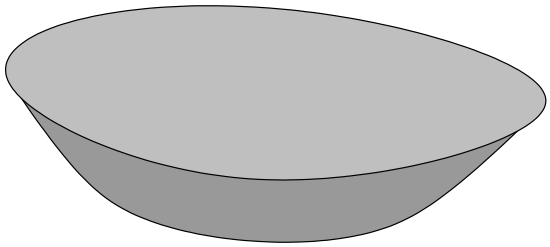
   (a) 51 (b) 43 (c) 61 (d) 56

   (e) 38 (f) 57 (g) 46 (h) 59

8. A 1/50 scale model of a pond is shown below. If the volume of the scale model is 4 cm^3, then the volume of the actual pond is

   (a) 1/2 m^3 (b) 50000 cm^3 (c) 500 m^3

   (d) not determined by the given information (e) none of the above



9. The remainder of 7^{100} divided by 9 is

   (a) 1 (b) 2 (c) 3 (d) 4

   (e) 5 (f) 6 (g) 7 (h) 8

10. How many distinct solutions x are there for the equation $\det(A - xI) = 0$ where det stands for determinant, I is the identity matrix, and A is the matrix

$$\begin{bmatrix} 1 & 0 & -3 & 0 \\ 0 & 2 & 0 & 1 \\ 1 & 0 & 3 & 0 \\ 1 & -2 & 0 & 1 \end{bmatrix}$$

    (a) no solution (b) 1 (c) 2 (d) 3 (e) more than 3

11. If $\omega^3 = 1$ and $\omega \neq 1$, find $\omega(\omega + 1)$.

    (a) i (b) $-i$ (c) 1 (d) -1 (e) $\left(-1 \pm \sqrt{3}\right)/2$

12. Which of the following is logically equivalent to (P or Q) and (Q or R)?

    (a) P and R (b) (P and R) or Q (c) (P and Q) or (Q and R)

    (d) (Not Q) and (P or R) (e) None of the above

13. When $x^{10}+1$ is divided by x^2-1, the remainder is

    (a) 1 (b) -1 (c) 0 (d) 2

    (e) none of these holds for every x

14. The equation $\sqrt{x+5}-\sqrt{x-2}+1=0$ has

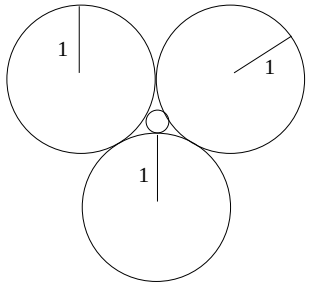
    (a) one real root (b) one real root and 1 complex root

    (c) 2 complex roots (d) two real roots (e) no root

15. The radius of the small circle in the middle is

    (a) $\frac{1}{6}$ (b) $\frac{1}{8}$ (c) $\frac{1}{4\sqrt{3}}$ (d) $\frac{\sqrt{3}-1}{4}$

    (e) $\frac{2\sqrt{3}-3}{3}$ (f) $\frac{2\sqrt{3}+1}{8}$ (g) $\frac{2\sqrt{3}}{9}$



16. Suppose α and β are the roots of $x^2+ax+b=0$. Then α^2 and β^2 are the roots of

    (a) $x^2+a^2x+b^2=0$

    (b) $x^2-a^2x+b^2=0$

    (c) $x^2+(2b-a^2)x+b^2=0$

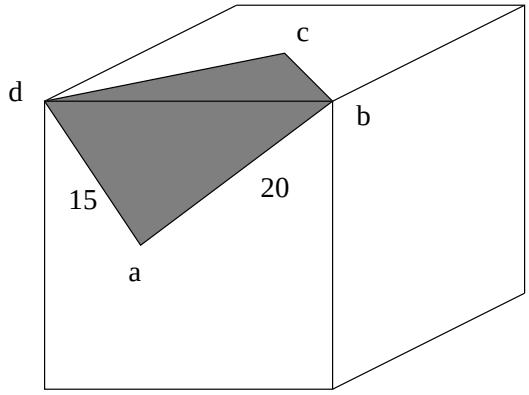
    (d) $x^2+(a^2-2b)x+b^2=0$

    (e) $x^2+2ax+b^2=0$

17. A number system based on 26 is used and the letters of the alphabet are the digits: $A=0$, $B=1$, $C=2$, $D=3,\cdots$, $X=23$, $Y=24$, $Z=25$. In this system, how much is ONE + ONE?

    (a) DBIA (b) BAID (c) BDAI (d) DAIB (e) BDIA

18. If a 15 by 20 rectangular piece of paper with corners a, b, c, and d is glued to the surface of a large cube so that the edge of the cube lies along the diagonal of the paper as shown, how far is corner a from corner c as measured through the cube?



(a) $13\sqrt{2}$ (b) 18 (c) 18.2 (d) $\sqrt{337}$

(e) None of the above

19. A function $f : X \to Y$ is called one-to-one if for $x_1, x_2 \in X$, $f(x_1) = f(x_2)$ implies $x_1 = x_2$. Define

$$\begin{aligned} f &: [-2, 0] \to \mathbb{R}, & f(x) &= \frac{x+1}{x-1} \\ g &: [0, 3] \to \mathbb{R}, & g(x) &= (x+1)^2 \\ h &: [-2, 1] \to \mathbb{R}, & h(x) &= \sqrt{x^2+1} \\ k &: [0, 3] \to \mathbb{R}, & k(x) &= (1-\sqrt{x})^2 \end{aligned}$$

which of the following is true.

(a) f and g are one-to-one functions

(b) g, h, k are one-to-one functions

(c) f, h, k are one-to-one functions

(d) f, k are one-to-one functions

(e) f, g, h, k are one-to-one functions

20. If C is a circle with center $\left(\sqrt{2}, \sqrt{3}\right)$, what is the maximum number of points (m, n) with integer coordinates that lie on C?

(a) 1 (b) 2 (c) 6 (d) 12 (e) no maximum