

2004 Georgia Tech High School Mathematics Competition
Junior Varsity Multiple Choice – Version A

1. $\sqrt{5/12} + \sqrt{1/5}$ $\sqrt{1/3} + \sqrt{2/7}$

What relation belongs in the box?

- (a) $<$ (b) $=$ (c) $>$ (d) $+$ (e) $-$

2. Solve for x : $3^x - 3^{-x} = 80/9$.

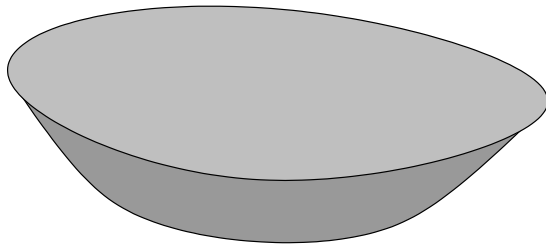
- (a) .8 (b) $\log_3\left(\frac{82}{19}\right)$ (c) $3/2$ (d) $\frac{40-\sqrt{1681}}{9}$ (e) 2

3. If m men can do a job in d days, then $m + r$ men can do the job in:

- (a) $d + r$ days (b) $d - r$ days (c) $\frac{md}{m+r}$ days
(d) $\frac{d}{m+r}$ days (e) none of these

4. A $1/50$ scale model of a pond is shown below. If the volume of the scale model is 4 cm^3 , then the volume of the actual pond is

- (a) $1/2 \text{ m}^3$ (b) 50000 cm^3 (c) 500 m^3
(d) not determined by the given information (e) none of the above



5. The remainder of 7^{100} divided by 9 is

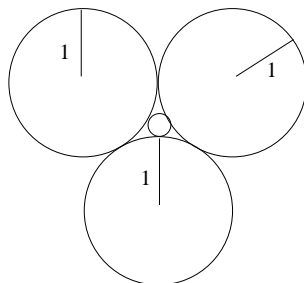
- (a) 1 (b) 2 (c) 3 (d) 4
(e) 5 (f) 6 (g) 7 (h) 8

6. In how many ways can one give change for \$2.83 using only pennies and nickels?
- (a) 51 (b) 43 (c) 61 (d) 56
 (e) 38 (f) 57 (g) 46 (h) 59
7. A drawer has 6 red socks and 6 white socks. If you reach in the drawer and randomly take out two socks, what is the chance (i.e., probability) that the two socks will match in color?
- (a) $\frac{5}{12}$ (b) $\frac{1}{2}$ (c) $\frac{2}{5}$ (d) $\frac{3}{7}$ (e) $\frac{5}{11}$ (f) $\frac{3}{8}$
8. $\sqrt{7 + 4\sqrt{3}} - \sqrt{7 - 4\sqrt{3}} =$
- (a) 4 (b) $2\sqrt{3}$ (c) $\sqrt{6}$ (d) 2 (e) $2\sqrt{7}$
9. Solve $|3x^2 - 2x - 1| \geq 0$
- (a) $x \geq 1$ (b) $x \leq -1/3$ (c) $x \leq -1$ or $x \geq 1$
 (d) $1 \leq x \leq -1/3$ (e) none of the above
10. If $\omega^3 = 1$ and $\omega \neq 1$, find $\omega(\omega + 1)$.
- (a) i (b) $-i$ (c) 1 (d) -1 (e) $(-1 \pm \sqrt{3})/2$
11. When $x^{10} + 1$ is divided by $x^2 - 1$, the remainder is
- (a) 1 (b) -1 (c) 0 (d) 2
 (e) none of these holds for every x

12. The radius of the small circle in the middle is

(a) $\frac{1}{6}$ (b) $\frac{1}{8}$ (c) $\frac{1}{4\sqrt{3}}$ (d) $\frac{\sqrt{3}-1}{4}$

(e) $\frac{2\sqrt{3}-3}{3}$ (f) $\frac{2\sqrt{3}+1}{8}$ (g) $\frac{2\sqrt{3}}{9}$



13. Suppose α and β are the roots of $x^2 + ax + b = 0$. Then α^2 and β^2 are the roots of

(a) $x^2 + a^2x + b^2 = 0$

(b) $x^2 - a^2x + b^2 = 0$

(c) $x^2 + (2b - a^2)x + b^2 = 0$

(d) $x^2 + (a^2 - 2b)x + b^2 = 0$

(e) $x^2 + 2ax + b^2 = 0$

14. The equation $\sqrt{x+5} - \sqrt{x-2} + 1 = 0$ has

(a) one real root (b) one real root and 1 complex root

(c) 2 complex roots (d) two real roots (e) no root

15. Which of the following is logically equivalent to $(P \text{ or } Q)$ and $(Q \text{ or } R)$?

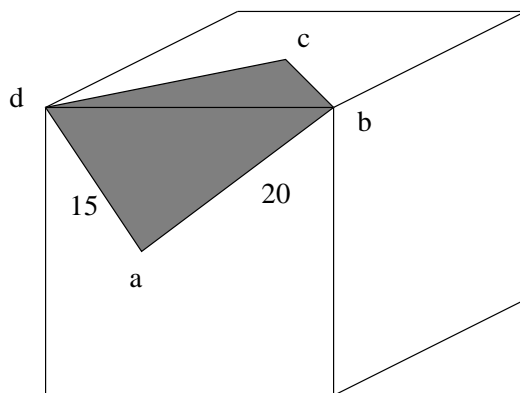
(a) P and R (b) $(P \text{ and } R)$ or Q (c) $(P \text{ and } Q)$ or $(Q \text{ and } R)$

(d) $(\text{Not } Q)$ and $(P \text{ or } R)$ (e) None of the above

16. How many distinct solutions x are there for the equation $\det(A - xI) = 0$ where \det stands for determinant, I is the identity matrix, and A is the matrix

$$\begin{bmatrix} 1 & 0 & -3 & 0 \\ 0 & 2 & 0 & 1 \\ 1 & 0 & 3 & 0 \\ 1 & -2 & 0 & 1 \end{bmatrix}$$

- (a) no solution (b) 1 (c) 2 (d) 3 (e) more than 3
17. If a 15 by 20 rectangular piece of paper with corners a , b , c , and d is glued to the surface of a large cube so that the edge of the cube lies along the diagonal of the paper as shown, how far is corner a from corner c as measured through the cube?



- (a) $13\sqrt{2}$ (b) 18 (c) 18.2 (d) $\sqrt{337}$
- (e) None of the above
18. If C is a circle with center $(\sqrt{2}, \sqrt{3})$, what is the maximum number of points (m, n) with integer coordinates that lie on C ?
- (a) 1 (b) 2 (c) 6 (d) 12 (e) no maximum

19. A function $f : X \rightarrow Y$ is called one-to-one if for $x_1, x_2 \in X$, $f(x_1) = f(x_2)$ implies $x_1 = x_2$. Define

$$\begin{aligned} f : [-2, 0] &\rightarrow \mathbb{R}, & f(x) &= \frac{x+1}{x-1} \\ g : [0, 3] &\rightarrow \mathbb{R}, & g(x) &= (x+1)^2 \\ h : [-2, 1] &\rightarrow \mathbb{R}, & h(x) &= \sqrt{x^2+1} \\ k : [0, 3] &\rightarrow \mathbb{R}, & k(x) &= (1-\sqrt{x})^2 \end{aligned}$$

which of the following is true.

- (a) f and g are one-to-one functions
 - (b) g, h, k are one-to-one functions
 - (c) f, h, k are one-to-one functions
 - (d) f, k are one-to-one functions
 - (e) f, g, h, k are one-to-one functions
20. A number system based on 26 is used and the letters of the alphabet are the digits: $A = 0, B = 1, C = 2, D = 3, \dots, X = 23, Y = 24, Z = 25$. In this system, how much is ONE + ONE?
- (a) DBIA (b) BAID (c) BDAI (d) DAIB (e) BDIA