

Georgia Institute of Technology High School Mathematics Competition

Ciphering Test

13 April 2019

Can you read this?

This is the font questions will be written in.

Instructions

- Ensure that you have 10 answer sheets. Write your ID number in the designated area NOW.
- Problems will be projected one at a time. You have 3 minutes per problem.
- You must write your final answer and question number in the designated areas of the answer sheets.
- When you have finished working, raise your answer sheet and a runner will collect it.
- The proctor will give a 10 second warning and call time at 3 minutes.
- When time is called you must put down your pencil immediately and raise your answer sheet for collection. NO late answers are accepted.
- Talking during a question will result in disqualification for the remaining portion of the cipher exam.

We are about to begin!

The first question will begin on the next slide.

Question 1

How many ways are there to permute the numbers 1, 2, 3, 4, 5, 6, 7 in such a way that 1 is to the left of 2 and 2 is to the left of 3 in the permutation? (Numbers are allowed to appear in between these three.)

Answer

$$7!/3! = 840$$

Question 2

How many integers can be represented as a sum of 1 or more integers between 1 and 20 (inclusive) without repeating summands.

Answer

210

Question 3

What are the three smallest whole numbers that cannot be written as a sum of 3 squares (including 0^2).

Answer

7, 15, 23

Question 4

Using Figure 1 below, find the area of the shaded isosceles triangle inside a quarter circle with radius 1.

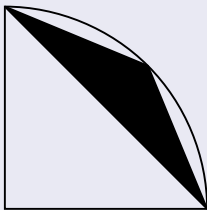


Figure: Isosceles triangle in a quarter circle.

Answer

$$\frac{\sqrt{2}-1}{2}$$

Question 5

Suppose that $p(x)$ is a polynomial whose coefficients are all non-negative integers where $p(1) = 8$ and $p(10) = 1034$. What is the polynomial p ?

Answer

$$p(x) = x^3 + 3x + 4$$

Question 6

At midnight the hour and minute hands of a clock are in alignment. After how many minutes will they next be in alignment again?

Answer

$$\frac{12(60)}{11} = \frac{720}{11} \text{ minutes}$$

Question 7

Find the sum of all real solutions to
 $(\log_2 x)^4 - 5(\log_2 x)^2 + 4 = 0.$

Answer

$$\frac{27}{4}$$

Question 8

What is the sum of all integers between 1 and 100 (inclusive) that do not contain a 3 as one of their digits?

Answer

4258

Question 9

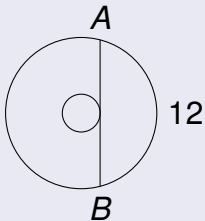
Suppose an alien visits Earth whose days are 28 hours long: 20 hours awake, 8 asleep. If the alien goes to bed at 10 pm on the first day, when would the alien wake up at the end of 100 28-hour cycles?

Answer

The alien wakes up at 10 pm.

Question 10

A circle C_1 has a chord AB of length 12 tangent to a concentric circle C_2 . Find the area of the washer between C_2 and C_1 .



Answer

36π

That's it!

Time for lunch!