TIE BREAKERS

0.1. **Problem #1.** Sum the infinite series:

$$\sum_{i=1}^{\infty} \frac{1}{(3i-2)(3i+1)}.$$

Solution. The answer is 1/3. We obtain a telescoping series where $S_n = \frac{1}{3}[(1-\frac{1}{4}) + (\frac{1}{4} - \frac{1}{7}) + \cdots + (\frac{1}{3n-2} - \frac{1}{3n+1})$. As n approaches ∞ the limit of this sum is 1/3.

0.2. **Problem #2.** How many integers between 2009 to 9002 are perfect squares?

Solution. The answer is 50. Notice that $45^2 = 2025$ and $95^2 = 9025$. So 45 is included and 95 is excluded.

0.3. **Problem #3.** Write x, y, and z in order from smallest to largest if $x = 2^{100}$, $y = 3^{75}$, and $z = 5^{50}$.

Solution. $x < z < y$.

0.4. **Problem #4.** Let $x_1, x_2, \ldots, x_7$ be seven distinct real numbers such that $\max_{i<j} |x_i - x_j| = 1$. Find the minimum value C such that for any such collection

$$\sum_{1 \le i < j \le 7} |x_i - x_j| < C.$$

Solution. 12 (In general, if there are $2k + 1$ points, $C = k^2 + k$, it is an limit that is never reached).

0.5. **Problem #5.** How many triples (p, q, r) are there such that p is a member of the set $\{1, 2, 3, 4, 5, 6\}$, q is a member of the set $\{7, 8, 9, 10, 11, 12\}$, r is a member of the set $\{13, 14, 15, 16, 17, 18\}$, and $p + q + r = 32$?

Solution. $15 = \binom{6}{2}$