GEORGIA TECH 2009 HIGH SCHOOL MATH COMPETITION PROBLEMS FOR ACTIVITIES–ANSWER KEY

PROBLEMS COMPILED BY MICHELLE DELCOURT

Problem #1

Special thanks to Zebediah Engberg of Hampshire College for suggesting this problem.

There are 20 prisoners in a jail. The warden announces that he has a puzzle, and if they can solve it, they will all be set free. The warden states that the prisoners will be placed in solitary cells. They will be unable to communicate with each other in any way. Down the hall, there is a room that contains a light bulb; the bulb is initially off. No one is able to see the light from inside his or her cell. Everyday, the warden picks a prisoner at random to enter the room with the light. While there, the prisoner can toggle the bulb if he or she wishes. The prisoner then has the option of claiming that all 20 prisoners have been in the room. If the prisoner is wrong, (that is, some still haven't been in the room), all 20 prisoners will be shot. However, if it is indeed true, all prisoners are set free. Therefore, the prisoner must be 100% certain.

The warden then lets the prisoners discuss plans. What is their strategy for survival?

Answer: One person is chosen to be the "counter". If the light is turned on, none of the other prisoners will touch it. If the light is off, and a prisoner who has never turned the light on enters, then he or she turns the light on. The light stays on until the counter returns and turns it off. It is important that the counter never turn the light on. Once the counter sees that the light has been turned on 19 times (he or she then knows that all the other prisoners have visited the room), he or she says that 20 prisoners have visited, and they can be freed.

Problem #2

Raymond Smullyan. The Riddle of Scheherazade. Harcourt Brace & Company, San Diego 1997

There are ten chests, and each chest has three drawers. Each of the thirty drawers contains either a diamond, an emerald, or a ruby. They are dispersed in the following manner:

1. D D D	6. D R R
2. D D E	7. E E R
3. D D R	8. E R R
4. D E E	9. E E E
5. D E R	10. R R R

You open any of the thirty drawers at random and find a diamond. Then you open another drawer of the same chest. What is the probability it contains a diamond?

Answer: There are 10 ways to find a diamond, and with each there are two ways

to pick a second drawer of the same chest. There are 20 ways to pick a second drawer. How many of these 20 contain a diamond. For Chests 4, 5, and 6 there are 0 ways. There are 2 ways in Chest 3 (two possible first diamonds and for each there is 1 way to pick a second diamond). Similarly there are 2 ways for Chest 2. For Chest 1 there are 3 possible first diamonds and 2 possible second diamonds for each of these, which makes 6 possibilities for Chest 1. There are 10 possible second diamonds the total 20 choices. Thus, the probability is 1/2.

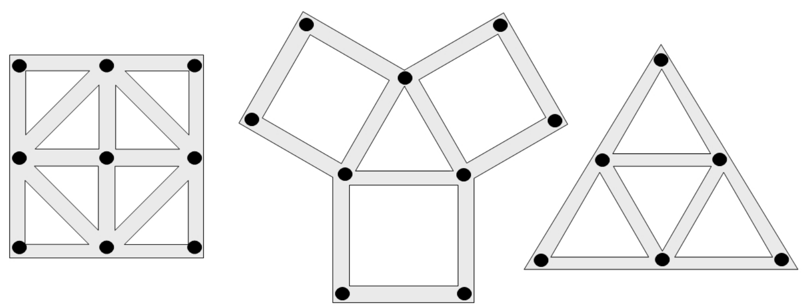
Problem #3

Special thanks to Alexandria Stephenson of Georgia Tech for suggesting this problem.

Ivan Moscovich. 1000 Play Thinks. Workman Publishing, New York 1997

Euler's Problem: the object of these puzzles is to trace the complete pattern marked out by the gray lines without picking up your pencil or backtracking over any sections. Your lines may cross only at the dots. Which of the following are solvable?

1.) 2.) 3.)



Answer: 2 and 3 are solvable, 1 is not

Problem #4

Special thanks to Kevin Bokelmann of Georgia Tech for suggesting this problem.

Observe the following:

$$\frac{64}{16} \rightarrow \frac{\not{6}4}{1\not{6}} = \frac{4}{1} = 4$$

Note: 64/16 = 4

Find another set of numbers with the property that when one crosses out the tens place of the numerator (a two digit number) and crosses out the ones place of denominator (another two digit number), the result is a fraction that reduces to the correct answer of the orginal fraction. The numbers crossed out must be the same. (11/11 = 1 does not count.)

Answer: Another solution is 95/19 = 5, if other solutions are presented, make sure the students have crossed out the tens place of the numerator and the ones place of denominator. Also check to make sure that the numbers crossed are the same and that the number which results is in fact the answer to the original fraction.

Problem #5

Boris A. Kordemsky. The Moscow Puzzles. Charles Scribner's Sons, New York 1992

There is an interesting five-digit number A. With a 1 after it, it is three times as large as with a 1 before it. What is it?

Answer: A with 1 after it is 10A + 1. With a 1 before it, it is 100,000 + A. Then 10A + 1 = 3(100,000 + A), and A = 42,857.

Problem #6

Raymond Smullyan. The Riddle of Scheherazade. Harcourt Brace & Company, San Diego 1997

Iskandar was an extremely intelligent person who once asked his friend Kamar the ages in years of his three children. The following conversation ensued:

Kamar: The product of their ages is 36.
Iskandar: That doesn't tell me their ages.
Kamar: Well, by coincidence, the sum of their ages is your own age.
Iskandar (after several minutes of thought): I still don't have enough information.
Kamar: Well, if this will help, my son is more than a year older than his two sisters.
Iskandar: Oh good! Now I know their ages.

What are their ages?
Answer: The following are the only triples with a product of 36 (their sums are written along side):

1, 1, 36–38	**1, 6, 6–13**
1, 2, 18–21	**2, 2, 9–13**
1, 3, 12–16	**2, 3, 6–11**
1, 4, 9–14	**3, 4, 3–10**

The only triples with the same sum are 1, 6, 6 and 2, 2, 9 (the sum is 13, Iskandar must be 13 because he could not determine the unique triple). Because the sum is more than a year older than both daughters, the correct solution is that both daughters are 2 and the son is 9.

Problem #7

Boris A. Kordemsky. The Moscow Puzzles. Charles Scribner's Sons, New York 1992

Scholars discovered 2,520 in hieroglyphs engraved on the stone lid of a tomb in an Egyptian pyramid. Why was such an honor paid this number?

Perhaps because it is divisible by every integer from 1 through 10. It is the lowest number so divisible. Demonstrate this.

Answer: The LCM (Least Common Multiple) is the product of distinct prime factors. The LCM of 1 through 10 is:
$2 \times 2 \times 2 \times 3 \times 3 \times 5 \times 7 = 2{,}520$.

Problem #8

Boris A. Kordemsky. The Moscow Puzzles. Charles Scribner's Sons, New York 1992

A horse travels half his route, with no load, at 12 miles per hour. The rest of the way a load slows him to 4 miles per hour.

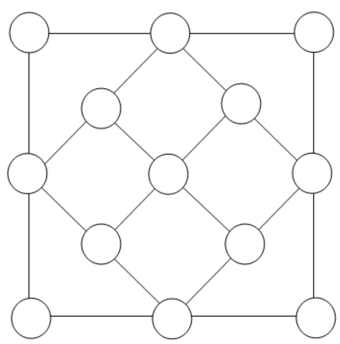
What is his average speed?

Answer: Consider the route taken as 1. The horse takes $1/2 \div 12 = 1/24$ unit of time for the first half, and $1/2 \div 4 = 1/8$ unit for the second half. The sum is $1/6$ unit, so the average speed is 6 miles per hour (8 is wrong)

Problem #9

Boris A. Kordemsky. The Moscow Puzzles. Charles Scribner's Sons, New York 1992

We have a crystal lattice whose "atoms" are joined in ten rows of 3 atoms each. Select thirteen integers, of which twelve are different, and place them in the "atoms" so that each row totals 20. (The smallest number needed is 1, and the largest number is 15.)



Answer: