Ciphering Round Varsity League

High School Math Competition 2008

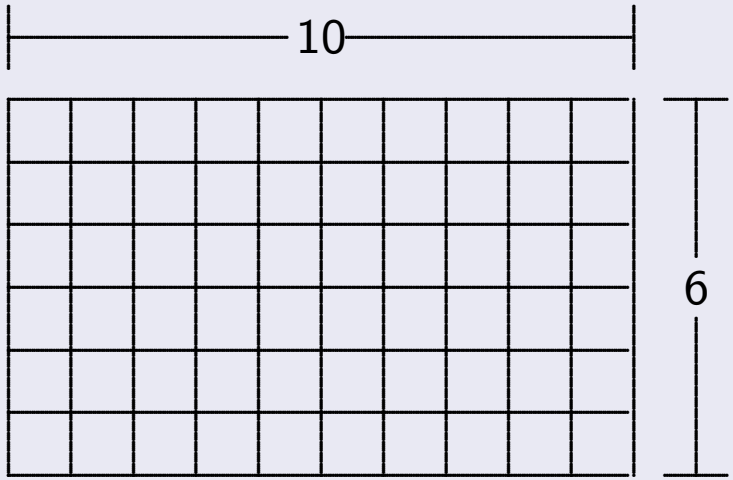
Georgia Institute of Technology

February 23rd, 2008

Problem #1

Problem

How many different squares of integer side length can be formed grouping together the cells (little squares in the grid) if each cell has side length 1.



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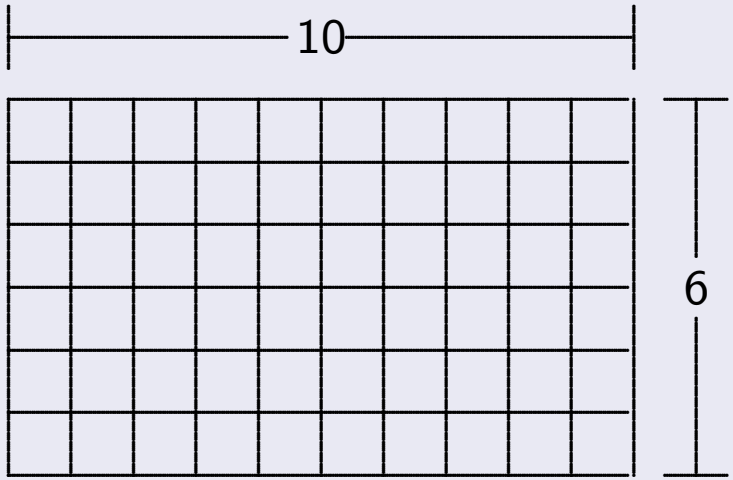

Answer

$$175 = 10 \cdot 6 + 9 \cdot 5 + 8 \cdot 4 + 7 \cdot 3 + 6 \cdot 2 + 5 \cdot 1$$

Problem #2

Problem

How many permutations of the name of the letters in the name

"*GEORGE P BURDELL*"

are there? (Ignore the spaces!)

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Answer

$$\frac{14!}{3!(2!)^3}$$

Problem #3

Problem

Find all the points (r, θ), with $0 \leq \theta \leq \pi$, where the curves $r^2 = \sin\theta$ and $r = 2 - \sin\theta$ intersect.

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Answer

$$\left(1, \frac{\pi}{2}\right)$$

Problem #4

Problem

Find the simplified value of

$$\frac{6508^2 - 2492^2}{4501^2 - 4499^2}.$$

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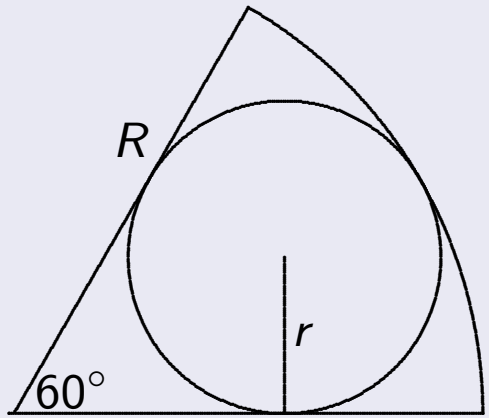
Answer

$$\frac{6508^2 - 2492^2}{4501^2 - 4499^2} = \frac{(6508 - 2492)(6508 + 2492)}{(4501 - 4499)(4501 + 4499)} = 2008$$

Problem #5

Problem

Γ is a circle with radius r. It is inscribed in a 60° sector of a circle with radius R. Find R/r.



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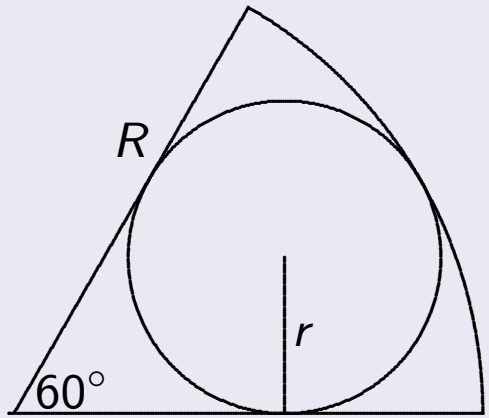

Answer

$$R/r = 3$$

Problem #6

Problem

Given that a, b, c and d are positive integers and that

$$a + \cfrac{1}{b + \cfrac{1}{c + \cfrac{1}{d}}} = \frac{181}{42},$$

find $a + b + c + d$.

Problem #6

Problem

Given that a, b, c and d are positive integers and that

$$a + \cfrac{1}{b + \cfrac{1}{c + \cfrac{1}{d}}} = \frac{181}{42},$$

find $a + b + c + d$.

Answer

$$4 + 3 + 4 + 3 = 14$$

Problem #7

Problem

If $a, b, 10, c, d$ are in geometric progression, find $abcd$.

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If $a, b, 10, c, d$ are in geometric progression, find $abcd$.

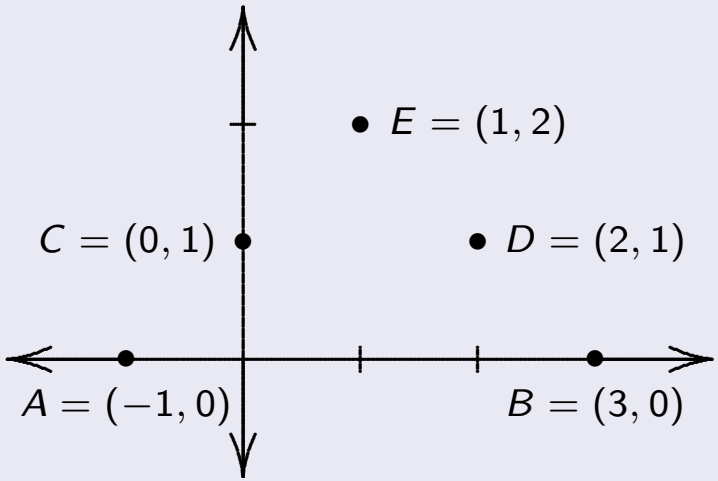
Answer

$$abcd = 10^4 = 10,000$$

Problem #8

Problem

Find the center of mass of the collection of points $\{A, B, C, D, E\}$ if A, C and E each have mass M and B and D each have mass $2M$



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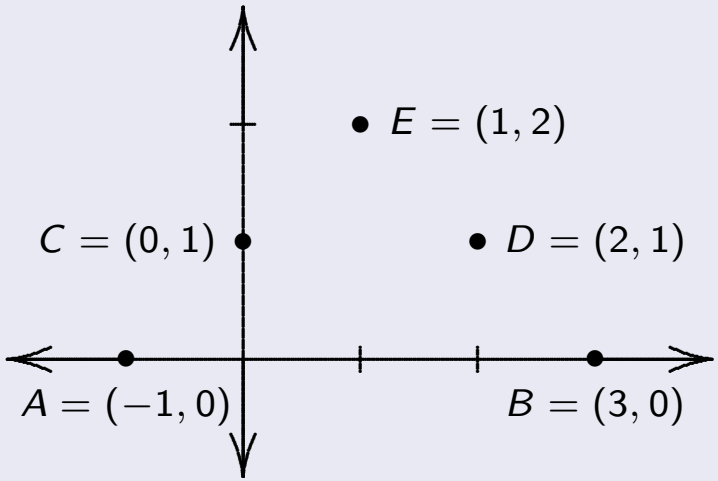

Answer

$$\left(\frac{10}{7}, \frac{5}{7}\right)$$

Problem #9

Problem

Of the 8-letter strings in which each letter is either A or B, how many contain the pattern $ABBA$ exactly once?

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Answer

70

Problem #10

Problem

Given the following binary numbers:

$$\begin{aligned} a_1 &= 101 \\ a_2 &= 1011 \\ a_3 &= 10111 \\ a_4 &= 101111 \\ a_5 &= 1011111 \\ a_6 &= 10111111 \end{aligned}$$

Evaluate $\sum_{k=1}^{6} a_k$, and express your answer in base 10.

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Evaluate $\sum_{k=1}^{6} a_k$, and express your answer in base 10.

Answer

372

$$\tilde{T}(h_\varepsilon) \quad e^{n_d}$$