Georgia Tech HSMC 2007

Varsity Multiple Choice

February 24th, 2007

1. In a recent competition, each of three teams played each other teams once. In the table, GF means *goals in favor* (the total number of goals scored by the team) and GA means *goals against* (the total number of goals scored against the team). What is the score of S vs J? (Give S's score first)

Team	Wins	Loses	Ties	GF	GA
S	1	0	1	6	4
F	1	0	1	3	2
J	0	2	0	2	5

(e)4-2

2. Legally married in Georgia[1], my neighbor has reached a square age. The product of the digits of his age is his wife's age. The age of their daughter is the sum of the digits of the father's age, and the age of their son is the sum of the digits of the mother's age. How old is the son?

(d) 9

3. A circular table is pushed into a corner in a rectangular room so that it touches both walls. A point on the edge of the table between the two points of contact is 2 inches from one wall and 9 inches from the other wall. What is the radius of the table?

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[1]To be legally married in Georgia, both spouses must be over 16 at the time of the wedding.

(d)17 inches.

4. How many three-element subsets of $\{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$ are there such that they contain at least two consecutive integers?

(d) 64

5. How many subsets of five elements of $\{1, 2, 3, 4, \ldots, 2007\}$ are there such that when the elements are placed in increasing order, they are in a geometric progression with integer ratio?

(d)160

6. What is $\operatorname{arccsc}\frac{5}{4} + \operatorname{arcsec}\frac{5}{4} + \arctan\frac{5}{4} + \operatorname{arccot}\frac{5}{4}$?

$(b)\pi$

7. What is the least positive integer greater than 1 such that if multiplied by 12912 gives a number that ends in 12912.

(c)6251

8. Which of the following 5 statements is true?

(d) Exactly 4 are false.

9. How many zeros are there at the end of $100!$?

$$\boxed{(e)24}$$

10. Let $ABCD$ be a unit square. Construct two circles: one with center A passing through C and the other with center C passing through A. Find the area of the region common to both circles.

$$\boxed{(b)\pi - 2}$$

11. If $P \subseteq \{1, 2, 3, \ldots, 49\}$ has the property that it does not contain two distinct elements which sum is divisible by 7. What is the maximal posible cardinality of P?

$$\boxed{(d)22}$$

12. Let $A_1 A_2 \ldots A_{100}$ be a regular polygon of 100 sides. How many isosceles triangles are there such that all their vertices are contained in the set $\{A_1, A_2, \ldots, A_{100}\}$

$$\boxed{(c)4900}$$

13. How many positive integers n satisfy that $((n!)!)!$ divides $(2007!)!$?

$$\boxed{(b)6}$$

14. If $\triangle ABC$ has sides with lengths 6, 7 and 8, find the value of $\cos \angle A + \cos \angle B + \cos \angle C$.

$$\boxed{(b)\frac{47}{32}}$$

15. An equilateral triangle and a regular hexagon have equal perimeters. What is the ratio of their areas?

$$\boxed{(d)\ \frac{2}{3}}$$

16. For $i = 1$ to 6, let $\log_a \left(\log_b (\log_c x_i) \right) = 0$, where a, b and c every possible different arrangement of 2,4,8. The product $x_1 x_2 x_3 x_4 x_5 x_6$ can be expressed in the form 2^N. Find N.

$$\boxed{(e)50}$$

17. The library of Winterland City has between 1000 and 2000 books. Of these, 25% are fiction, 1/13 or them are biographies and 1/17 of them are Atlases. How many books are either biographies or atlases?

$$\boxed{(a)240}$$

18. Simplify $\sqrt[3]{2 + \sqrt{5}} + \sqrt[3]{2 - \sqrt{5}}$.

$$\boxed{(a)1}$$

19. Find the sum of the absolute values of all the fix points[2] of

$$g(x) = (x^2 - 4x + 3)^2 - 4(x^2 - 4x + 3) + 3.$$

$$\boxed{(c)8}$$

20. The solution of the equation $\log_2 \log_4 x + \log_4 \log_2 x = 2$ is:

$$\boxed{(e)S = \{16\}}$$

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[2]A fix point of a function f is a value of x for which $f(x) = x$