

Ciphering Round Varsity League

High School Math Competition 2007

Georgia Institute of Technology

February 24th, 2007

Problem #1

Problem

Calculate the sum in closed form of

$$\frac{1}{2!} + \frac{2}{3!} + \frac{3}{4!} + \cdots + \frac{n}{(n+1)!}.$$

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$$\frac{1}{2!} + \frac{2}{3!} + \frac{3}{4!} + \cdots + \frac{n}{(n+1)!}.$$

Answer

$$1 - \frac{1}{n!}$$

Problem #2

Problem

If C_1 and C_2 are concentric circles and ℓ is a line intersecting C_1 at A and B , and is tangent to C_2 . If $AB = 2$. What is the area of the area of the annular region between the circles?

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Answer

$$\pi$$

Problem #3

Problem

After a typist had written ten letters and had addressed the ten corresponding envelopes, a careless mailing clerk inserted the letters in the envelopes at random, one letter per envelope. What is the probability that exactly nine letters were inserted in the proper envelope?

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Answer

0

Problem #4

Problem

How many negative roots does the equation $x^4 - 5x^3 - 4x^2 - 7x + 4 = 0$ have?

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How many negative roots does the equation $x^4 - 5x^3 - 4x^2 - 7x + 4 = 0$ have?

Answer

None

Problem #5

Problem

Suppose that p is strictly 0 and 15. For values of x between p and 15 (inclusive), find the smallest value of $f(x)$ is given by

$$f(x) = |x - p| + |x - 15| + |x - p - 15|.$$

Problem #5

Problem

Suppose that p is strictly 0 and 15. For values of x between p and 15 (inclusive), find the smallest value of $f(x)$ is given by

$$f(x) = |x - p| + |x - 15| + |x - p - 15|.$$

Answer

15

Problem #6

Problem

Which is larger, $\sqrt[8]{8!}$ or $\sqrt[9]{9!}$?

Problem #6

Problem

Which is larger, $\sqrt[8]{8!}$ or $\sqrt[9]{9!}$?

Answer

$$\sqrt[8]{8!} < \sqrt[9]{9!}$$

Problem #7

Problem

Find an integer value of n such that

$$\sqrt{\frac{5n-2}{n+3}}$$

is also an integer.

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Find an integer value of n such that

$$\sqrt{\frac{5n-2}{n+3}}$$

is also an integer.

Answer

$$n = 14$$

Problem #8

Problem

Find the value of $\cos^4 \frac{\pi}{24} - \sin^4 \frac{\pi}{24}$.

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Problem

Find the value of $\cos^4 \frac{\pi}{24} - \sin^4 \frac{\pi}{24}$.

Answer

$$\begin{aligned}\cos^4 \frac{\pi}{24} - \sin^4 \frac{\pi}{24} &= \frac{\sqrt{6} + \sqrt{2}}{4} \\ &= \frac{\sqrt{\sqrt{3} + 1}}{2}\end{aligned}$$

Problem #9

Problem

Bob and Alice are standing next to each other, each holding a sign with three statements written upon it. Bob's sign says

- (a) Exactly one of her statements is wrong.
- (b) Exactly two of her statements are wrong.
- (c) Exactly three of her statements are wrong.

On the other hand, Alice's sign says:

- (a) Exactly one of his statements is right.
- (b) Exactly two of his statements are right.
- (c) Exactly three of his statements are right.

Exactly how many true statements are there, total, on the two signs?

Problem #9

Problem

Bob and Alice are standing next to each other, each holding a sign with three statements written upon it. Bob's sign says

- (a) Exactly one of her statements is wrong.
- (b) Exactly two of her statements are wrong.
- (c) Exactly three of her statements are wrong.

On the other hand, Alice's sign says:

- (a) Exactly one of his statements is right.
- (b) Exactly two of his statements are right.
- (c) Exactly three of his statements are right.

Exactly how many true statements are there, total, on the two signs?

Answer

2

Problem #10

Problem

Let $f(x) = ax - b|x|$, where a and b are uniformly and independently chosen randomly between -1 and 1 . Find the probability that f is onto.

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Answer

$$\frac{1}{2}$$

THE END