# Georgia Tech HSMC 2007

## Junior Varsity Multiple Choice

## February $24^{th}$, 2007

1. Legally married in Georgia[1], my neigbor has reached a square age. The product of the digits of his age is his wife's age. Their daughter's age is the sum of the digits of the father's age, and their son's age is the sum of the digits of the mother's age. How old is the son?

$\boxed{(d)\ 9}$

2. In the following cryptogram, each letter represents a distinct decimal digit,

$$7(FRYHAM) = 6(HAMFRY)$$

$\boxed{(E) M = 8}$

3. How many subsets of four elements of $\{1, 2, 3, 4, \ldots, 100\}$ are there such that when the elements are placed in increasing order, they are in a geometric progression with integer ratio.

$\boxed{(c) 16}$

4. A right triangle has hypotenuse $c$ and semiperimeter $s$, then, the area is equal to:

$\boxed{(d) s(s-c)}$

5. In the square $ABCD$, the point $E$ is between $A$ and $B$ and the poitn $F$ is between $B$ and $C$. Find $\angle AEF + \angle EFC$.

$\boxed{(c) 270^\circ}$

6. When I am as old as my father is now, I will be five times as old as my son is now. By then, my son will be eight years older than I am now. The sum of my father's age and my age is 100 years. How much older am I than my son?

$\boxed{(d)\ 22 \text{ years.}}$

7. How many digits are used to number the pages of a book having 100 pages numbered 1 to 100?

$\boxed{(c)\ 192}$

8. Which of the following 5 statements is true?

$\boxed{(d) \text{ Exactly 4 are false.}}$

9. How many numbers between 10 and 1000 have the property that their digits are in strictly increasing order? (ex:125)

$\boxed{(b) 120}$

[1] To be legally married in Georgia, both spouses must be over 16 at the time of the wedding.

10. Let $r$ and $s$ be the solution to the equatio $x^2 + 3x + c = 0$, If $r^2 + s^2 = 33$, find the value of $c$.

$\boxed{(b)\ -12}$

11. In two regular polygons, it is known that the measure of the internal angle of one of them exceeds the measure of the internal angle of the other by $20°$. Furthermore when the measure of the external angle of one of them is added to the measure of the external angle of the other one obtains $100°$. What is the addition of the number of sides of both polygons?

$\boxed{(b)15}$

12. A circular table is pushed into a corner in a rectangular room so that it touches both walls. A point on the edge of the table between the two points of contact is 2 inches from one wall and 9 inches from the other wall. What is the radious of the table?

$\boxed{(d)17 \text{ inches.}}$

13. Chris traveled 1 hour longer and 2 miles farther than Calvin, but averaged 3mph slower. If the sum of their times was 4 hours, what is the sume, in miles, of the distance they traveled?

$\boxed{(d)24}$

14. Which of the following is NOT a factor of $x^4 - 4x^3 - x^2 + 16x - 12$?

$\boxed{(d)x + 1}$

15. How many prime factors does 8027 have?

$\boxed{(b)2}$

16. What is the least positive integer greater than 1 such that if multiplied by 616 it yields a number that ends in 616?

$\boxed{(d)126}$

17. How many different four-digit numbers can be formed by arranging the digits 2, 0, 0 and 7?

$\boxed{(a)6}$

18. Let $A_1A_2A_3A_4A_5A_6A_7A_8$ be a regular octagon. How many right triangles are there such that all their vertices are contained in the set $\{A_1, A_2, A_3, A_4, A_5, A_6, A_7, A_8\}$.

$\boxed{(d)24}$

19. Suppose $f(x) = ax + b$ and $g(x) = bx + a$, with $a$ and $b$ are positive integers. if $f\left(g(50)\right) - g\left(f(50)\right) = 28$, find the product $ab$.

$\boxed{(b)\ 12}$

20. A date is called *weird* if the number of the month and the number of the day have 1 as the greatest common factor. What is the fewest number of *weird* days in any month?

$\boxed{(b)\ 10}$