Ciphering Round Junior Varsity League

High School Math Competition 2007

Georgia Institute of Technology

February 24^{th}, 2007

Problem #1

Problem

How many pairs of integers (m, n) are there such that $mn = m + n$?

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Answer

1

Problem #2

Problem

If a certain number is reduced by 7 and the remainder is multiplied by 7, the result is the same as if the number is reduced by 11 and the remainder is multiplied by 11. What is the number?

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Answer

$$18 = 7 + 11$$

Problem #3

Problem

If x and y are distinct positive real numbers, which ratio is greater,

$$\frac{x^2+y^2}{x+y} \text{ or } \frac{x^2-y^2}{x-y}?$$

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Answer

$$\frac{x^2+y^2}{x+y} \leq \frac{x^2-y^2}{x-y}$$

Problem #4

Problem

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Answer

8 kids

Problem #5

Problem

How many positive integer solutions (a, b, c) are there to the equation

$$2004^a + 2005^b = 2006^c?$$

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How many positive integer solutions (a, b, c) are there to the equation

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Answer

None (one side is odd, the other is even)

Problem #6

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A new edition of a book is published every seven years. When the seventh edition was issued, the sum of the publication years was 13524. When was the book first published?

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Answer

1926

Problem #7

Problem

There is a real number $x > 1$ such that, if I add it to its reciprocal, and then I add what I get to *its* reciprocal, I end up with the sum equal to 27. Find $\lfloor x \rfloor$, the integer part of x.

Problem #7

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There is a real number $x > 1$ such that, if I add it to its reciprocal, and then I add what I get to *its* reciprocal, I end up with the sum equal to 27. Find $\lfloor x \rfloor$, the integer part of x.

Answer

26

Problem #8

Problem

Find a function $f(x) \neq x$ such that for every $x \geq 0$,

$$f\left(\frac{x}{1+x}\right) = \frac{f(x)}{1+f(x)}.$$

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Answer

One is $f(x) = \dfrac{x}{1+x}$

Problem #9

Problem

If x, y and z are non-zero real numbers such that $x - y = xy = \frac{x}{y} = z$, find all possible values of z.

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Answer

$$z = \frac{1}{2}$$

Problem #10

Problem

Some friends are trying to divide a pile of stones. They try to split the stones into three equal groups, but end up with 1 stone left over. Surprisingly, the same thing happens when the try splitting the stones into equal groups of 4, 5, 6, and 7; there is always 1 stone left over! What is the fewest number of stones that the friends could have (beyond just 1 stone of course!)?

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Answer

421

THE END