

Georgia Tech HSMC 2006

Junior Varsity Multiple Choice

Answers

1. How many different prime factors has the number $2^{20} - 2^{17} + 14$?

(A) 3

2. Find the *smallest* integer $n > 10$ such that the fraction $\frac{71}{7n+1}$ is reducible.

(D) 81

3. The graph of a second degree polynomial f intersects that of $g(x) = x^2$ at $x = -1, 0, 1$. Compute the smallest possible value of $f(x)$.

(B) 0

4. Triangle ABC has $|AB| = 5$, $|BC| = 6$, and $|AC| = 7$. A circle is inscribed in the triangle, intersecting AB at D . Find $|AD|$.

(D) 3

5. Suppose that x and y are real numbers that satisfy $\lfloor x \rfloor + \{y\} = 3.2$, and $\{x\} + \lfloor y \rfloor = 1.7$, where $\lfloor x \rfloor$ represents the greatest integer less than or equal to x , and $\{x\} = x - \lfloor x \rfloor$. Find the difference $x - y$.

(D) 2.5

6. How many functions $f : \mathbb{R} \rightarrow \mathbb{R}$ satisfy, for all real x, y , $f(x) + f(y) = xf(y)$

(B) 1

7. There are 7 students in David's *Linear and Discrete Mathematics* class. Knowing there are 3 students who major in electrical engineering, 4 students who major in mathematics, and 1 student who double majors in both electrical engineering and mathematics, how many students in David's class major in neither electrical engineering nor mathematics?

(B) 1

8. How many unordered pairs $\{a, b\}$ can one select from $\{1, 2, \dots, 50\}$ such that $|a - b| = 5$?

(A) 45

9. How many pairs (a, b) of positive integer numbers are solution of the equation $a^2 - b^2 = 35$?

(C) 2

10. If x, y, c are real numbers satisfying $|x| \leq c$ and $|x - y| \leq 1$, find the largest possible value of $|y^2 - x^2|$.

(C) $2c + 1$

11. How many anagrams of the word *georgiatech* are there?

(C) $\frac{11!}{2! \cdot 2!}$

12. Let S_1 be a square and C_1 be a circle inscribed in S_1 . Let S_2 be a square inscribed in C_1 and let C_2 be a circle inscribed in S_2 . Let A_1 be the area of the region bounded between S_1 and C_1 , and let A_2 be the area of the region bounded between S_2 and C_2 . Compute $\frac{A_1}{A_2}$.

(D) 2

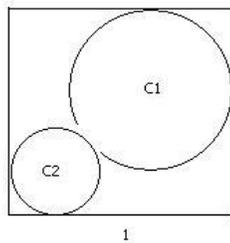
13. A rectangular box having integer dimensions (in meters) greater than 1 has volume $2006ft^3$. What is its surface area in ft^2 ?

(D) 2310

14. A regular octahedron has volume $V \neq 1$. Its surface area can be written as $S = 2^a 3^b V^c$, where a, b, and c are rational numbers. Find $a + b + c$.

(A) $\frac{5}{2}$

15. Given a square with sidelength 1, determine the radius of the smaller circle C_2 given C_1 's radius is twice that of C_2 's, and these circles are tangent to each other and the square.



(B) $\frac{\sqrt{2}}{3(\sqrt{2} + 1)}$

16. Given a set of 7 different points in the plane, how many segments can we form such that each of those segments have both of its end points in the set.

(B) 21

17. For all real numbers a, b and some fixed real number n , the operation $*$ is defined as $a * b = a^2 - b * n$. Given that $11 * 11 = 5$, what is the value of $20 * 3$?

(B) 396

18. How many values $-\infty < x < +\infty$ satisfies $x[x] = 8$, where $[x]$ denotes the largest integer less than or equal to x ?

(B) 1

19. A 3-digit palindrome is a 3-digit number (not starting with 0) which reads the same backwards and forwards (for example, 171). Find the sum of all even 3-digit palindromes.

(B) 22000

(A) 20200 (B) 22000 (C) 22450
(D) 45450 (E) 49500

20. How many pairs (a, b) of positive integer numbers are solution of the equation

$$a^2 + b^2 = 338.$$

(D) 3