

Ciphering Round Junior Varsity League

High School Math Competition 2006

Georgia Institute of Technology

March 4th, 2006

Problem #1

Problem

In the cryptosum

$$\begin{array}{rcccc}
 & & & & S & E & N & D \\
 + & & & & M & O & R & E \\
 \hline
 & M & O & N & E & Y & &
 \end{array}$$

What should be the value of M?

Problem #1

Problem

In the cryptosum

$$\begin{array}{rcccc} & & S & E & N & D \\ + & & M & O & R & E \\ \hline M & O & N & E & Y & \end{array}$$

What should be the value of M?

Answer

$$M = 1$$

Problem #2

Problem

If $x = 1 + 2^p$ and $y = 1 + 2^{-p}$, find y in terms of x only.

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If $x = 1 + 2^p$ and $y = 1 + 2^{-p}$, find y in terms of x only.

Answer

$$y = 1 + \frac{1}{x-1} = \frac{x}{x-1}$$

Problem #3

Problem

The addition of two numbers is 12 and the product is 30. What is the addition of the inverses of those two numbers?

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Answer

$$\frac{2}{5}$$

Problem #4

Problem

What is the least number of children in a family if each child has at least one brother and at least one sister?

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Answer

4

Problem #5

Problem

If $a * b = \frac{a + b + 3}{5ab}$, compute $(1 * 1) * (1 * 1)$.

Problem #5

Problem

If $a * b = \frac{a + b + 3}{5ab}$, compute $(1 * 1) * (1 * 1)$.

Answer

1

Problem #6

Problem

Find the least positive integer number divisible by 11 such that the sum of its digits is divisible by 11.

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Find the least positive integer number divisible by 11 such that the sum of its digits is divisible by 11.

Answer

209

Problem #7

Problem

Find the sum of the digits of $10^{100} - 101$.

Problem #7

Problem

Find the sum of the digits of $10^{100} - 101$.

Answer

899

Problem #8

Problem

How many numbers between 10 and 1000 have the property that their digits are in strictly increasing order?

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Problem

How many numbers between 10 and 1000 have the property that their digits are in strictly increasing order?

Answer

120

Problem #9

Problem

What is the last digit (unit digit) of $3^{1001} \cdot 7^{1002} \cdot 13^{1003}$?

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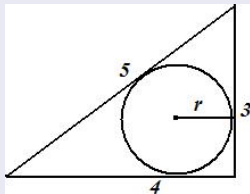
Answer

9

Problem #10

Problem

In the following figure:

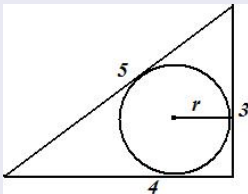


What is the value of the radius r ?

Problem #10

Problem

In the following figure:



What is the value of the radius r ?

Answer

$$r = 1$$

THE END