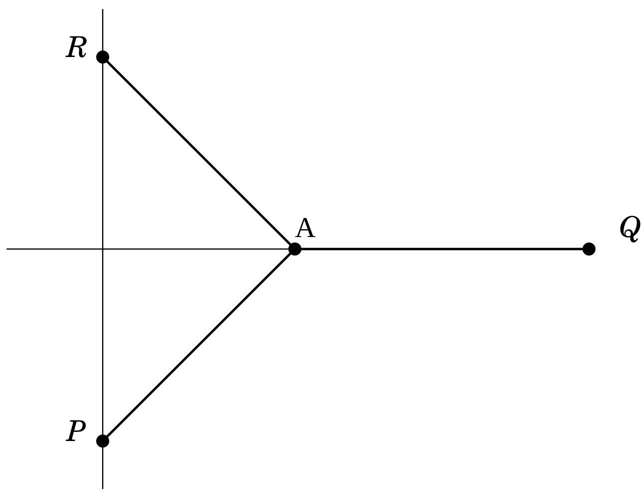
2004 Georgia Tech High School Mathematics Competition
Varsity Proof

1. If three pieces of string are used to connect three pegs $P = (0, -1)$, $Q = (2, 0)$, and $R = (0, 1)$ to a fourth (movable) peg $A = (a, 0)$, what is the least length of string that can be used? (Do not count the string used for tying knots or tying to the pegs.)



2. Show that for any number $n = 1, 2, 3, \ldots$ there is a circle C on the xy-plane with center $\left(\sqrt{2}, \sqrt{3}\right)$ having exactly n points (j, k) with integer coordinates inside (but not on) C.

3. Prove that there exist infinitely many pairs of positive integers x, y with
$$x(x+1) \mid y(y+1), \qquad \text{but}$$
$$x \nmid y, \qquad x+1 \nmid y, \qquad x \nmid y+1, \qquad x+1 \nmid y+1.$$
Find the pair x and y satisfying the conditions above and for which $x + y$ is as small as possible.

4. A positive integer a is called *tight* if given any $2a - 1$ positive integers, it is always possible to choose exactly a of them whose sum is divisible by a. Show that if a is tight and b is tight, then ab is tight.

5. Let a be a 3 digit number. Arrange the digits of a in the order that results in a 3 digit number which is as small as possible. Call that number b. Again rearrange the digits of a to obtain a 3 digit number that is as large as possible. Call that number c. If $b - c = a$. Find a.