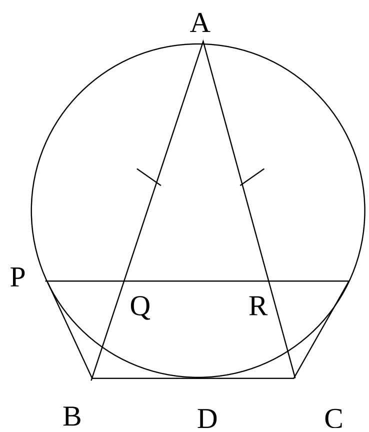
**2004 Georgia Tech High School Mathematics Competition**
Junior Varsity Proof

1. Show that $(a+b+c)(bc+ab+ac) \geq 9abc$ for all positive numbers $a$, $b$, and $c$.

2. Let $\triangle ABC$ be isoceles with $AB = AC$. Let $K$ be the circle tangent to $\overline{BC}$ at its midpoint $D$ and passing through $A$. Draw the other tangets to $K$ from $B$ and $C$ and connect the resulting points of tangency to form a segment with endpoint $P$ that intersects $\triangle ABC$ at points $Q$ and $R$. Find the ratio of $PQ$ to $QR$.




3. Find the center of a circle of radius $r > 0$ that meets every circle passing through $(-1, 0)$ and $(1, 0)$ at $90°$ angles.

4. If $n_1 + n_2 + \cdots + n_k = n$ and $n_1, \ldots, n_k$ are integers between 1 and $n$ inclusive, then $\{n_1, \ldots, n_k\}$ is called a *partition of* $n$ with *partition elements* $n_1, \ldots, n_k$. (Note that $1+2$ and $2+1$ are the same partitions of 3.) How many partitions are there of $n$ with the maximum difference between parition elements equal to 1 or 0.

5. Let $a_1, a_2, \ldots, a_n$ be the interior angles (measured in radians) of a polygon with $n$ sides. Similarly, let $b_1, \ldots, b_m$ be the interior angles of a polygon with $m$ sides. If $m > n$ and $s = (a_1 - b_1) + (a_2 - b_2) + \cdots + (a_n - b_n) - b_{n+1} - b_{n+2} - \cdots - b_m$, prove $(m-n)\pi + s = 0$.