**2004 Georgia Tech High School Mathematics Competition**
Junior Varsity Proof Solution

jv#1.

$$\begin{aligned}(a+b+c)(bc+ab+ac) &= abc + a^2b + a^2c + b^2c + ab^2 + bc^2 + ac^2 \\ &= 3ab + b(a^2+c^2) + c(a^2+b^2) + a(b^2+c^2) \\ &\geq 3ab + b(2ac) + c(2ab) + a(2bc) \\ &= 9abc.\end{aligned}$$

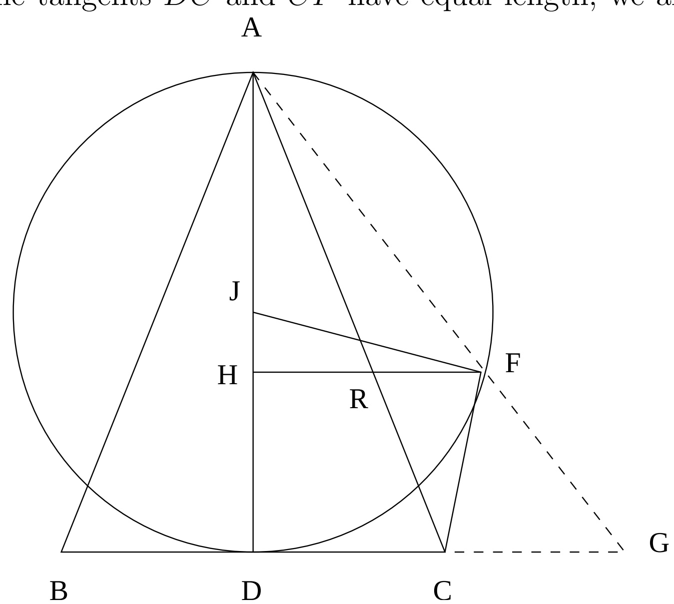
**2004 Georgia Tech High School Mathematics Competition**
Junior Varsity Proof Solution

jv#2. Let $H$ be the intersection of $AD$ and $QR$. We will show that $HR = RF$ where $F$ is the other intersection of the line through $PR$ with the circle. Thus the answer is $1 : 2$.

Draw the segment from the center of the circle $J$ to $F$ and the line from the top of the circle $A$ through $F$ until it intersects the extension of $BC$ in a point $G$.

By similar triangles, it is enough to show that $DC = CG$.

Let $t$ be the measure of angle $AFJ$ which is the same as angle $JAF$ since triangle $JAF$ is isoceles. Computing around the vertex $F$ and using the fact that angle $JFC$ is right, we see that angle $CFG$ measures $\pi/2 - t$. On the other hand, triangle $ADG$ is right, so angle $FGC$ measures $\pi/2 - t$ as well. Thus, triangle $CFG$ is isoceles, and we have that $CG = CF$. Since the tangents $DC$ and $CF$ have equal length, we are done.



**2004 Georgia Tech High School Mathematics Competition**
Junior Varsity Proof Solution

jv#3. First we observe that two circles intersect at right angle is equivalent to $r_1^2 + r_2^2 = |c_1 c_2|^2$. Now, all circles passing through $(-1, 0)$ and $(1, 0)$ are having the form

$$C_1 : x^2 + (y - a)^2 = a^2 + 1$$

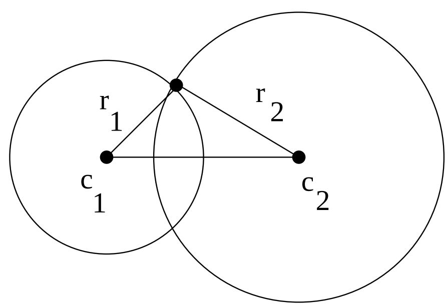
with center at $(0, a)$ and radius $\sqrt{a^2+1}$. Consider a circle $C_2 : (x - b)^2 + y^2 = r^2$ with center $(b, 0)$. If $C_1$ and $C_2$ intersect at right angle, we need

$$\begin{aligned} a^2 + 1 + r^2 &= (\text{distance between } (b, 0) \text{ and } (0, a))^2 \\ &= a^2 + b^2 \end{aligned}$$

so

$$b = \pm\sqrt{1 + r^2}$$

So the center of the desired circle is $\pm\sqrt{1 + r^2}$.



**2004 Georgia Tech High School Mathematics Competition**
Junior Varsity Proof Solution

jv#4. The partition problem: Here one is asked to find the number of partitions of an integer $n$ using two consecutive positive integers. Call the number of such partitions $f(n)$. One can write each partition uniquely as

$$i_1 + \cdots + i_k + j_1 + \cdots + j_\ell$$

where the $i$'s are all one more than the $j$'s and there may be no $j$'s, but there must be some $i$'s. Setting aside the partition $1 + 1 + \cdots + 1$ ($n$ times), we can consider the partition

$$i_1 + \cdots + i_{k-1} + (i_k - 1) + j_1 + \cdots + j_\ell$$

which is an acceptable partition of $n - 1$. It is easily checked that this correspondence is one to one and onto. Hence, there is exactly one more acceptable partition of $n$ than of $n - 1$, namely, the one we set aside with $n$ ones. Thus, $f(n) = f(n - 1) + 1$. Since $f(1) = 1$, we see that $f(n) = n$.

**2004 Georgia Tech High School Mathematics Competition**
Junior Varsity Proof Solution

jv#5.

$$\begin{aligned}(m-n)\pi + s &= m\pi - (b_1 + \cdots + b_m) - n\pi + (a_1 + \cdots + a_n)\\ &= [(\pi - b_1) + \cdots + (\pi - b_m)] - [(\pi - a_1) + \cdots + (\pi - a_n)]\\ &= 2\pi - 2\pi\\ &= 0.\end{aligned}$$

The second rearrangement of terms expresses the quantity as the difference of the sums of the exterior angles of the two polygons. The exterior angles of any polygon sum to $2\pi$.